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A GENERALIZED THERMOELASTIC MEDIUM SUBJECTED TO PULSED LASER HEATING VIA A TWO-TEMPERATURE MODEL

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This article investigates stress and induced temperature in an isotropic, homogeneous, thermoelastic half-space using a two-temperature generalized thermoelasticity model. The bounding plane surface of the present half-space continuum is subjected to a non-Gaussian laser pulse. Laplace's transform space is considered to deduce a closed-form solution to the problem. In addition, the inversions of Laplace's transformations have been carried numerically to obtain field quantities in the transient state. The effects of parameters of two-temperature, laser-pulse and laser intensity are investigated. A concluding remark for the graphical forms of the derived expressions is presented.

Keywords: two-temperature model; thermoelasticity, non-Gaussian laser pulse; laser intensity

1. Introduction

The one-relaxation theory of generalized thermoelasticity has been introduced by Lord and Shulman (LS) (1967). The heat equation of LS theory is of the wave-type to ensure finite speeds of propagation for heat and elastic waves. In fact, this theory eliminates the paradox inherent in other classical thermoelasticity theories. These theories predict infinite speed of propagation of heat waves contrary to physical observations. Two additional generalizations to the coupled classical theory of thermoelasticity are presented. Müller (1971) considered some restrictions on the constitutive equations by proposing entropy production inequality which is not enough and need to be modified. Green and Laws (1972) proposed a generalization of Müller's inequality. Moreover, Green and Lindsay (GL) (1972) obtained an explicit version of constitutive equations. In fact, the GL theory contained two constants that acted as relaxation times. These thermal relaxations modified not only the heat equation but also other governing equations of the coupled thermoelasticity theory. In other way, Tzou (1995) produced a dual-phase-lags thermoelasticity theory which is considered as another generalization to the coupled thermoelasticity one.

The two-distinct-temperature model is formulated by Chen and Gurtin (1968), Chen and Williams (1968) and Chen *et al.* (1969). There is no difference between two temperatures in the absence of heat supply in the time-independent situation (Chen *et al.*, 2004). While in the presence of the heat supply, this difference is proportional to it. However, for the time-dependent situation, these temperatures are different regardless the presence or absence of the heat supply. The two temperatures are related to strain in the case of a traveling wave plus throughout the medium (Boley and Tolins, 1962). Warren and Chen (1973) discussed the wave propagation problem via a two-temperature model of thermoelasticity. Additional investigation about the two-temperature model is given by Zenkour and Abouelregal (2014a). In addition, Abouelregal and Zenkour (2017) used the two-temperature model to deal with some problems in the half--space or a semi-infinite solid induced by pulsed laser heating and the micropolar thermoelastic media. Recently, Zenkour (2018) presented a refined two-temperature multi-phase-lags theory for the thermomechanical response of microbeams.

The laser-induced vibrations and thermoelastic wave induced by pulsed laser heating was the subject of many investigators (Welsh *et al.*, 1988; Wang and Xu, 2001; Sun *et al.*, 2008; Kumar *et al.*, 2015). Recently, Al-Lehaibi (2016) studied the induced temperature and stress fields in an elastic infinite medium with a cylindrical cavity under the two-temperature theory. Moreover, Allam and Tayel (2017) investigated the thermoelastic behavior of a semi-infinite medium heated uniformly by a laser beam having temporally Gaussian distribution.

The present work used the two-temperature generalized thermoelasticity model to investigate thermal field quantities in an elastic half-space. A non-Gaussian laser beam with pulse duration of two picoseconds heats the bounding plane surface. The Laplace transform space is used to obtain an analytical solution of the problem. The inverse Laplace transforms are numerically presented using the Riemann sum approximation method. Numerical estimations of two temperatures, displacement, stress and strain distributions are discussed. The derived field quantities are numerically obtained and the results are graphically presented. Effects due to the parameters of two-temperature, laser-pulse and laser intensity are investigated.

2. Basic equations

The governing equations are represented according to the classical linear dynamical thermoelasticity theory as

$$\begin{aligned}
&K\varphi_{,ii}=\rho C_E\Big(1+\tau_0\frac{\partial}{\partial t}\Big)\frac{\partial\theta}{\partial t}+\gamma T_0\Big(1+\tau_0\frac{\partial}{\partial t}\Big)\frac{\partial e_{ij}}{\partial t}-\Big(1+\tau_0\frac{\partial}{\partial t}\Big)Q\\
&(\lambda+\mu)u_{i,ij}+\mu u_{i,jj}-\gamma\theta_{,i}=\rho\ddot{u}_i
\end{aligned}\tag{2.1}$$

where $e_{ij}=(u_{i,j}+u_{j,i})/2$ denotes the strain tensor in which u_i represent components of the displacement vector, $\gamma=(3\lambda+2\mu)\alpha_t$, in which λ and μ denote Lamé's coefficients and α_t is linear thermal expansion. The thermodynamical temperature is denoted by $\theta=T-T_0$ in which T_0 represents the reference temperature. In addition, Eqs. (2.1) includes such parameters as K which denotes thermal conductivity, C_E – specific heat at constant strain, Q – heat source, τ_0 is thermal relaxation time, ρ is material density, and φ denotes conductive temperature that satisfies the expression

$$\varphi-\theta=b\varphi_{,ii}\tag{2.2}$$

where $b>0$ is the parameter of two-temperature. It is to be noted that as $b\to 0$ and $\varphi\to\theta$, the 1TT is recovered.

The constitutive equations are given by

$$\sigma_{ij}=2\mu e_{ij}+\lambda e_{kk}\delta_{ij}-\gamma\theta\delta_{ij}\tag{2.3}$$

where σ_{ij} represent stress components.

3. Formulation of the problem

Consider a homogeneous isotropic thermoelastic conducting solid occupying the half-space $x\geqslant 0$. It obeys Eqs. (2.1)-(2.3) without considering body forces and magnetization. The present half--space is uniformly irradiated on the bounding plane $x=0$ by a laser pulse with a non-Gaussian temporal profile. The system is initially quiescent, i.e. the state functions are temperature--independent.

The displacement field for the present one-dimensional medium can be represented by $u_x = u(x,t)$ and $u_y = u_z = 0$. So, $e = e_{xx} = \partial u/\partial x$ represents the unique strain component. In addition, the heat conduction equation is written as

$$K\frac{\partial^2 \varphi}{\partial x^2} = \Big(1 + \tau_0 \frac{\partial}{\partial t}\Big)\Big[\frac{\partial}{\partial t}(\theta + \gamma T_0 e) - Q\Big] \tag{3.1}$$

and the constitutive equation is simplified to be

$$\sigma_{xx} = \sigma = (\lambda + 2\mu)e - \gamma\theta \tag{3.2}$$

The equation of motion is represented as

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial \theta}{\partial x} = \rho\ddot{u} \tag{3.3}$$

or may appear as

$$\frac{\partial^2 \sigma}{\partial x^2} = \rho\frac{\partial^2 e}{\partial t^2} \tag{3.4}$$

The two temperatures are related to each other by

$$\varphi - \theta = b\frac{\partial^2 \varphi}{\partial x^2} \tag{3.5}$$

To simplify the governing equations of the problem, we used the dimensionless variables

$$\begin{aligned} &x' = c_1\eta x \qquad &&u' = c_1\eta u \qquad &&\tau_0' = c_1^2\eta\tau_0 \qquad &&t' = c_1^2\eta t \\ &\theta' = \frac{\gamma\theta}{\rho c_1^2} \qquad &&\varphi' = \frac{\gamma\varphi}{\rho c_1^2} \qquad &&\sigma' = \frac{\sigma}{\rho c_1^2} \qquad &&Q' = \frac{Q}{K c_1^2\eta^2 T_0} \end{aligned} \tag{3.6}$$

where $c_1 = \sqrt{\lambda + 2\mu/\rho}$ denotes longitudinal wave speed and $\eta = \rho C_E/K$ denotes thermal viscosity. Hence, one gets

$$\begin{aligned} &\frac{\partial^2 \varphi}{\partial x^2} = \Big(1 + \tau_0\frac{\partial}{\partial t}\Big)\Big(\frac{\partial \theta}{\partial t} + \varepsilon\frac{\partial e}{\partial t} - Q\Big) \\ &\sigma = e - \theta \qquad \frac{\partial^2 \sigma}{\partial x^2} = \frac{\partial^2 e}{\partial t^2} \qquad \varphi - \theta = \beta\frac{\partial^2 \varphi}{\partial x^2} \end{aligned} \tag{3.7}$$

where

$$\varepsilon = \frac{\gamma^2 T_0}{\rho^2 C_E c_1^2} \qquad \beta = b c_1^2\eta^2 \tag{3.8}$$

Let us suppose that the medium is uniformly heated by a laser pulse with a non-Gaussian form temporal profile (Sun *et al.*, 2008; Zenkour and Abouelregal, 2014b)

$$I(t) = \frac{L_0 t}{t_p^2}\exp\Big(\frac{-t}{t_p}\Big) \tag{3.9}$$

where t_p denotes the characteristic time of the laser-pulse, L_0 represents laser intensity which is defined as the total energy carried by a laser pulse per unit area of the laser beam (Sun *et al.*, 2008). So, the energy heat source $Q(x,t)$ near the surface is given by (Sun *et al.*, 2008)

$$Q(x,t) = \frac{1-R}{\delta}\exp\Big(\frac{x - h/2}{\delta}\Big)I(t) = \frac{R_a L_0}{\delta t_p^2}t\exp\Big(\frac{x - h/2}{\delta} - \frac{t}{t_p}\Big) \tag{3.10}$$

in which δ denotes absorption depth of heat energy and R_a denotes surface reflectivity. When $x = 0$, the laser pulse lies on the surface of the medium and then the energy source is reduced to be a function of dimensionless time as

$$Q(t) = \frac{R_a L_0}{\delta t_p^2}t\exp\Big(\frac{-h}{2\delta} - \frac{t}{t_p}\Big) \tag{3.11}$$

4. Solution in Laplace's transform domain

It is well known from now that the Laplace transform is defined by

$$f(s)=\int\limits_0^\infty f(t)\mathrm{e}^{-st}\,dt \tag{4.1}$$

If it is applied to Eqs. (3.7), we obtain differential equations

$$\begin{aligned}&\frac{d^2\overline{\varphi}}{dx^2}=s(1+\tau_0 s)\overline{\theta}+\varepsilon s(1+\tau_0 s)\overline{e}-\overline{G}(s)\\ &\frac{d^2\overline{\sigma}}{dx^2}=s^2\overline{e}\qquad \overline{\sigma}=\overline{e}-\overline{\theta}\qquad \overline{\theta}=\overline{\varphi}-\beta\frac{d^2\overline{\varphi}}{dx^2}\end{aligned} \tag{4.2}$$

where

$$\overline{G}(s)=\frac{\varepsilon_2 t_p^2(1+\tau_0 s)}{(1+st_p^2)^2}\qquad \varepsilon_2=\frac{\gamma R_a L_0}{Kc_1\delta t_p^2}\mathrm{e}^{-h/2\delta} \tag{4.3}$$

Eliminating $\overline{\theta}$ from Eqs. (4.2), one obtains

$$\Big(\frac{d^4}{dx^4}-A\frac{d^2}{dx^2}+B\Big)\{\overline{\varphi},\overline{e}\}=\{-F(s),0\} \tag{4.4}$$

where

$$\begin{aligned}&A=\frac{s^2(1+\beta)+\alpha_1(1+\varepsilon)}{1+\beta(1+\alpha_1\varepsilon)}\qquad B=\frac{s^2\alpha_1}{1+\beta(1+\alpha_1\varepsilon)}\\ &F(s)=\frac{s^2\overline{G}(s)}{1+\beta(1+\alpha_1\varepsilon)}\end{aligned} \tag{4.5}$$

The solutions to Eq. (4.4) take the following forms

$$\overline{\varphi}=-\frac{F(s)}{B}+A_1\mathrm{e}^{-m_1}+A_2\mathrm{e}^{-m_2}\qquad \overline{e}=B_1\mathrm{e}^{-m_1}+B_2\mathrm{e}^{-m_2} \tag{4.6}$$

where A_i and B_i are some parameters given in terms of s. From Eqs. (4.2), one can get the following relation

$$B_i=-\frac{\beta m_i^4-m_i^2}{m_i^2-s^2}A_i=\varOmega_i A_i\qquad i=1,2 \tag{4.7}$$

Thus, one gets

$$\overline{e}=\varOmega_1 A_1\mathrm{e}^{-m_1}+\varOmega_2 A_2\mathrm{e}^{-m_2} \tag{4.8}$$

Substituting Eqs. (4.6) and (4.8) into Eqs. (4.2), one obtains

$$\begin{aligned}&\overline{\theta}=-\frac{F(s)}{B}+(1-\beta m_1^2)A_1\mathrm{e}^{-m_1}+(1-\beta m_2^2)A_2\mathrm{e}^{-m_2}\\ &\overline{\sigma}=\frac{F(s)}{B}+(\varOmega_1-1+\beta m_1^2)A_1\mathrm{e}^{-m_1}+(\varOmega_1-1+\beta m_2^2)A_2\mathrm{e}^{-m_2}\end{aligned} \tag{4.9}$$

The two parameters A_1 and A_2 can be given after applying the boundary conditions on the boundary plane $x=0$.

4.1. Thermal boundary condition

At $x = 0$, we have $\varphi(0,t) = 0$ or (after applying the Laplace transformation) $\overline{\varphi}(0,s) = 0$.

4.2. Mechanical boundary condition

Also, at $x = 0$, we consider the traction-free case, then $\sigma(0,t) = 0$ or (after applying the Laplace transformation) $\overline{\sigma}(0,s) = 0$.

The above boundary conditions at $x = 0$ give the parameters A_1 and A_2 as

$$\{A_1, A_2\} = \frac{F(s)}{B[\Omega_1 - \Omega_2 + \beta(m_1^2 - m_2^2)]}\{-\Omega_2 - \beta m_2^2, \Omega_1 + \beta m_1^2\} \tag{4.10}$$

Finally, the solution in the Laplace transform domain may be written as

$$\begin{aligned}
\overline{\varphi} &= -\frac{F(s)}{B}\Big[1 + \frac{(\Omega_2 + \beta m_2^2)\mathrm{e}^{-m_1} - (\Omega_1 + \beta m_1^2)\mathrm{e}^{-m_2}}{\Omega_1 - \Omega_2 + \beta(m_1^2 - m_2^2)}\Big] \\
\overline{e} &= -\frac{F(s)[\Omega_1(\Omega_2 + \beta m_2^2)\mathrm{e}^{-m_1} - \Omega_2(\Omega_1 + \beta m_1^2)\mathrm{e}^{-m_2}]}{B[\Omega_1 - \Omega_2 + \beta(m_1^2 - m_2^2)]} \\
\overline{\theta} &= -\frac{F(s)}{B}\Big[1 + \frac{(1 - \beta m_1^2)(\Omega_2 + \beta m_2^2)\mathrm{e}^{-m_1} - (1 - \beta m_2^2)(\Omega_1 + \beta m_1^2)\mathrm{e}^{-m_2}}{\Omega_1 - \Omega_2 + \beta(m_1^2 - m_2^2)}\Big] \\
\overline{\sigma} &= -\frac{F(s)}{B}\Big[1 + \frac{(\Omega_1 - 1 + \beta m_1^2)(\Omega_2 + \beta m_2^2)\mathrm{e}^{-m_1} - (\Omega_2 - 1 + \beta m_2^2)(\Omega_1 + \beta m_1^2)\mathrm{e}^{-m_2}}{\Omega_1 - \Omega_2 + \beta(m_1^2 - m_2^2)}\Big]
\end{aligned} \tag{4.11}$$

So, the solution in Laplace's transformation domain is completely obtained.

5. Numerical results

All functions in the Laplace domain should be inverted to the time domain through the sum

$$f(t) = \frac{\mathrm{e}^{\zeta t}}{t}\Big\{\frac{1}{2}\overline{f}(\zeta) + \mathrm{Re}\Big[\sum_{n=1}^{N}(-1)^n \overline{f}\Big(\zeta + \frac{\mathrm{i}n\pi}{t}\Big)\Big]\Big\} \tag{5.1}$$

where i is the imaginary number unit and Re is used for the real part. For faster convergence, numerical experiments have shown that the value of ζ that satisfies the above relation is given in terms of time as $\zeta \approx 4.7/t$ (Tzou, 1996).

The formula given in Eq. (5.1) is used to invert the Laplace transforms in Eqs. (4.11). The field quantities are represented graphically along the axial direction. The material properties of copper are considered in the numerical examples. These constants at $T_0 = 293\,\mathrm{K}$ are given by: $K = 368\,\mathrm{N/(Ks)}$, $\alpha_t = 1.78 \cdot 10^{-5}\,\mathrm{K}^{-1}$, $C_E = 383.1\,\mathrm{m^2/K}$, $\rho = 8954\,\mathrm{kg/m^3}$, $\lambda = 7.76 \cdot 10^{10}\,\mathrm{N/m^2}$, $\mu = 3.86 \cdot 10^{10}\,\mathrm{N/m^2}$.

Computations are carried out along the x-axis with $0 \leqslant x \leqslant 1$ for a small value of dimensionless time $t = 0.2$. The following constants are assumed for the computation purpose, $R_a = 0.5$, $h = 0.1$, $\delta = 0.01$ and $\tau_0 = 0.02$. In addition, the laser intensity is given by $L_0 = c \cdot 10^{11}\,\mathrm{J/m^2}$ where c is the laser intensity parameter.

Results are presented for two cases. The first one is to investigate how the non-dimensional conductive temperature, thermodynamic temperature, displacement, stress, and strain vary with different values of dimensionless temperature discrepancy β. Note that $\beta = 0$ indicates the one-dimensional temperature theory (1TT) as the old situation while $\beta = 0.2$ or 0.4 represents the 2TT as new situations. In this case, one assumes that the characteristic time of laser-pulse is $t_p = 2$ picoseconds and the laser intensity parameter $c = 1$. Figure 1a shows that the

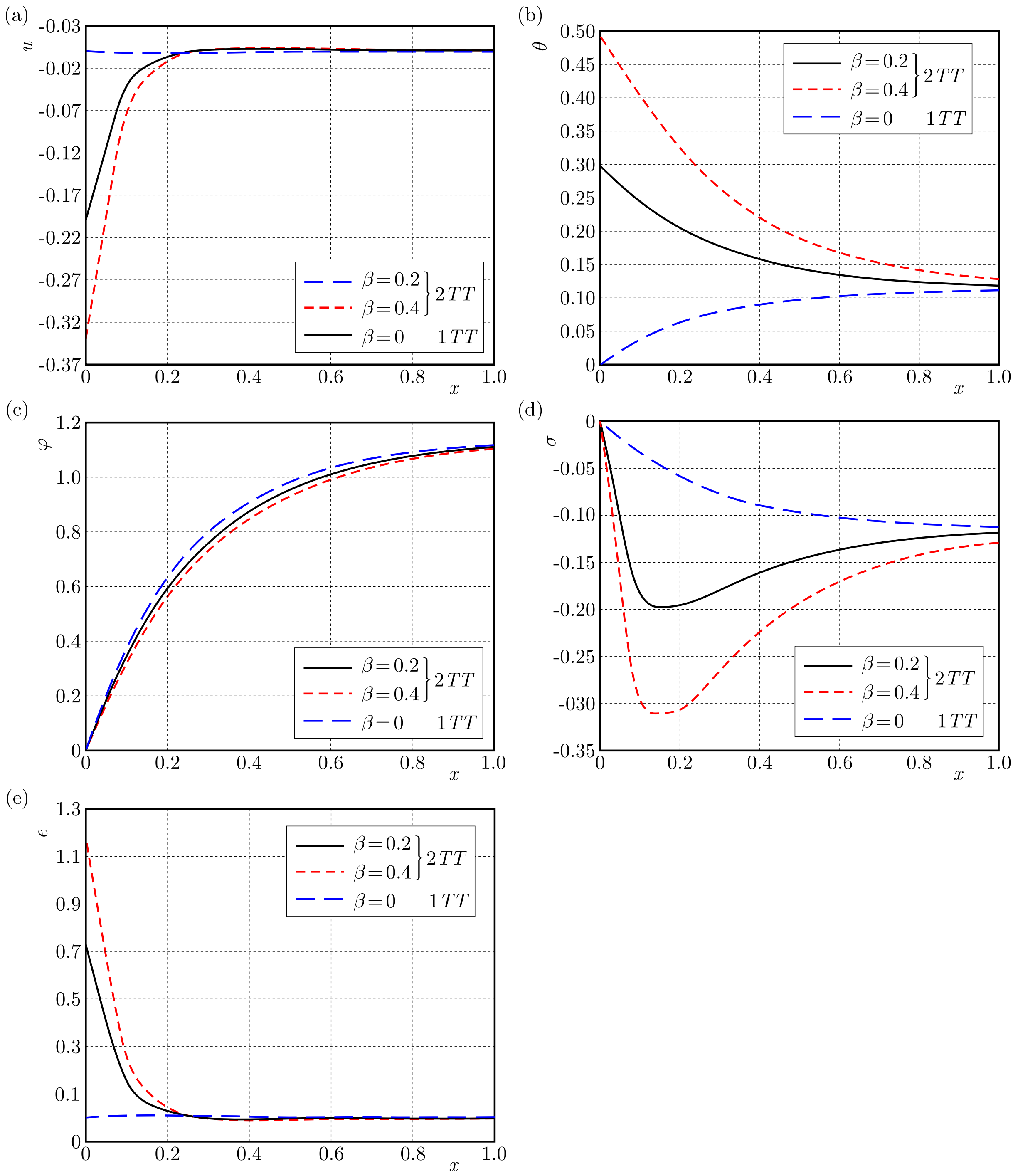

Fig. 1. Dependence of (a) displacement u, (b) thermodynamical temperature θ, (c) conductive temperature φ, (d) thermal stress σ, and (e) strain e on the two-temperature parameter β

displacement u increases along the x-axis for the two-temperature generalized thermoelasticity theory (2TT) with $\beta = 0.2$ and $\beta = 0.4$ while it decreases for the old situation (1TT of L-S with $\beta = 0$). It is to be noted that β has no effect on the displacement u for $x = 0.6$. Figure 1b shows that the thermodynamical temperature θ increases as β increases. The relative error between the results may decrease as x increases. Figure 1c shows that the conductive temperature φ increases along the x axis and decreases as β increases. It starts from the zero value and terminates at different values according to the value of β. Figure 1d plots the stress σ along the x-axis. It decreases with an increase of x and β. Finally, Fig. 1e shows that the strain e decreases along the x-axis for the two-temperature generalized thermoelasticity $\beta = 0.2$ and $\beta = 0.4$ while it increases for the old situation ($\beta = 0$). It is to be noted that β has no effect on the strain e for $x = 0.6$. These figures show that β has a significant effect on all the fields.

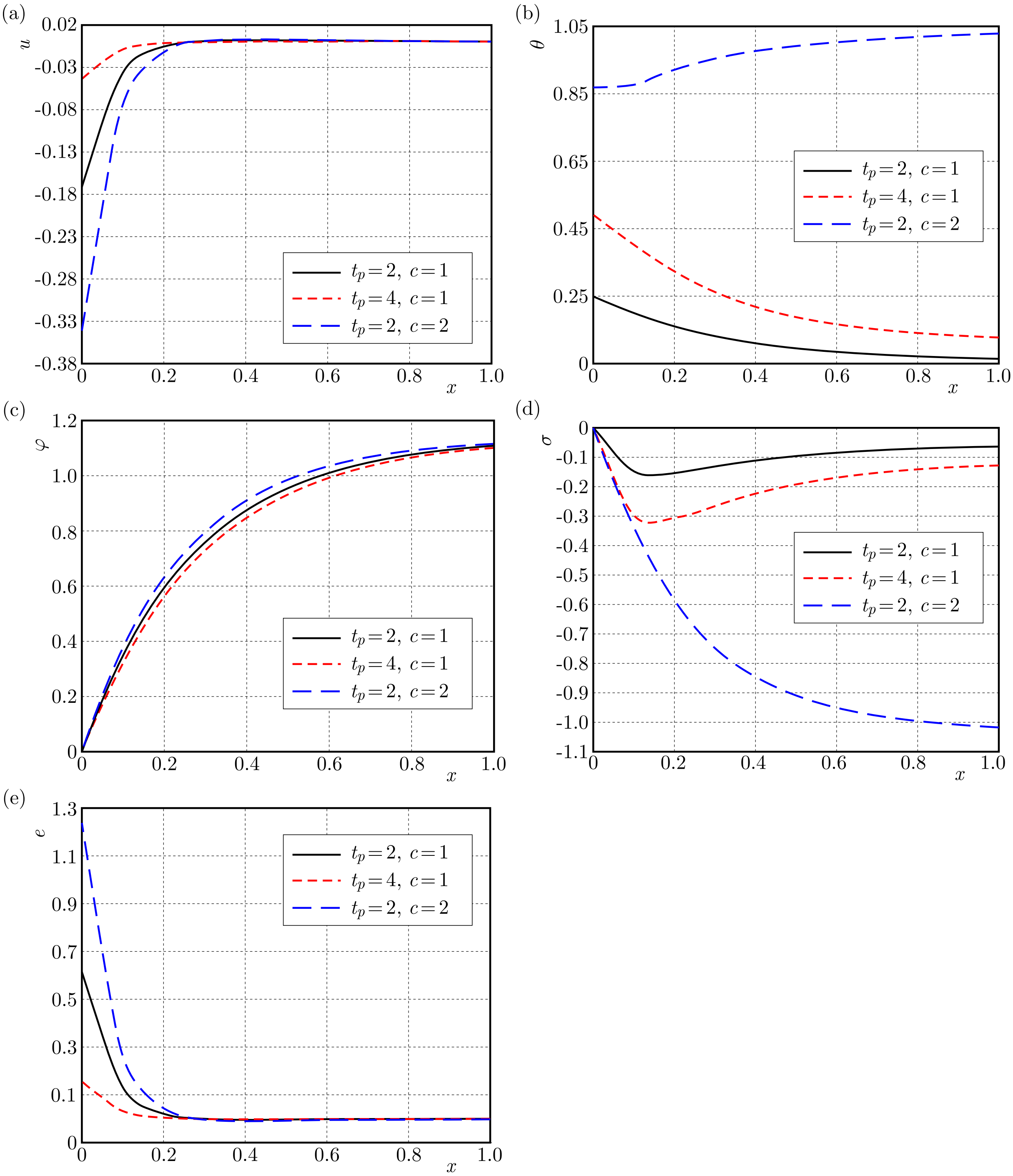

Fig. 2. Dependence of (a) displacement u, (b) thermodynamical temperature θ, (c) conductive temperature φ, (d) thermal stress σ, and (e) strain e on the time of the laser-pulse and the laser intensity

The second case is to investigate how the results vary with variation of t_p and c in the case when the temperature discrepancy parameter remains constant $\beta = 0.4$. From Fig. 2, it is found that the characteristic time of the laser-pulse t_p and laser intensity parameter c have significant effects on the behavior of field quantities. For example, the displacement and stress are decreasing with an increase of t_p at a fixed value of c and with an increase of c at a fixed value t_p. However, the thermodynamic temperature, conductive temperature and strain are increasing with an increase of t_p at a fixed value of c and with an increase of c at a fixed value t_p. In addition, the displacement and both temperatures are increasing along the x-axis while the stress and the corresponding strain are decreasing.

6. Conclusions

The present article investigates all fields in a thermoelastic half-space. A non-Gaussian laser beam with pulse duration of two picoseconds is used to heat the surface of the half-space. Governing equations are presented in the context of the model of two-temperature generalized thermoelasticity. The Laplace transform technique is used to obtain the exact forms of conductive temperature, thermodynamical temperature, stress, strain and displacement distributions in the transformed domain. Effects of temperature discrepancy as well as laser-pulse and laser intensity parameters on the field variables are investigated.

The results show that the two-temperature parameter plays a significant role in the behavior of all field variables. In addition, it is found that the laser-pulse and the laser intensity parameters have significant influence. The paper indicates that the model of two-temperature generalized thermoelasticity presented herein describes the behavior of the particles of an elastic medium more realistically than the one-temperature generalized thermoelasticity model.

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